sensor or the first state of the sensor; ("), to second sensor or second state of the same sensor; ($\bar{\ }$), relative dimensionless quantity or function.

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NONDESTRUCTIVE MONITORING METHODS IN THE INVESTIGATION OF THE THERMOPHYSICAL CHARACTERISTICS OF SOLIDS

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Nondestructive methods are proposed for the complex determination of the thermophysical characteristics of solids on the basis of solutions of a system of two-dimensional nonstationary heat-conduction equations. Appropriate computational formulas are presented.

In recent years, a number of papers [1-3] has been published whose authors use the regularities of two- and three-dimensional nonstationary temperature field development in a half-space to determine the thermophysical characteristcs of substances when heat is supplied through a circle of known radius.

If the methods of determining the thermal properties which are based on the two-dimensional nonstationary solutions of the classical boundary-value problem of heat conduction for a half-space are compared with the corresponding one-dimensional methods [4], the deduction can be made that the principal advantage of the former is the possibility of executing complex measurements of the thermal diffusivity, thermal conductivity, and thermal activity coefficients of solids for known values of the temperature and heat flux on just the body surface in its local heating area. Therefore, to find the thermophysical characteristics mentioned from one experiment, there is no need to spoil the wholeness of the specimen and install appropriate sensors therein. Moreover, because of the reduction in the time to prepare the specimen for the experiment, the productivity of the method is raised significantly.

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The essential complexity of the computational nature in determining a, b, and λ must be noted when the two-dimensional nonstationary methods are used since even for particular axisymmetric cases of temperature field development, these latter are described by special parabolic—cylinder and Whittaker functions and probability integrals [5-9]. The difficulty mentioned can be avoided by using modern computer engineering facilities.

Solutions of the heat-conduction equation for discontinuous homogeneous or mixed boundary conditions are the theoretical basis for the two-dimensional methods. Thus, for a stationary heat-conduction equation with a circular interfacial line for the boundary conditions of the first kind on the half-space surface, we arrive at a Dirichlet problem, while for a circular interfacial line for the boundary conditions of the second kind we have a Newmann problem.

When boundary conditions of the first or third kind are given on the half-space surface within the heating spot and boundary conditions of the second kind outside it (or conversely), we have mixed boundary conditions. In this case we must deal with the appropriate solutions of the so-called dual integral equations of mathematical physics [10].

Let us write the general formulation of the classical boundary-value problem of heat conduction for a half-space with heat supplied through a circle of known radius.

We have a semibounded body whose initial temperature is constant, equal to T_0, at all points. On the surface bounding the half-space ($z = 0$, $0 \leqslant r < \infty$), a discontinuous boundary condition of the second kind is given:

a) A specific heat flux $q(\tau)$ is delivered through a circle of radius $r = r_0$ (the origin is selected at the center of the circle);

b) There is no heat flux outside the circle ($r > r_0$).

Find the temperature distribution $T_1(r, z, \tau)$ in the domain $0 \leqslant r < r_0$, $z \geqslant 0$, $\tau > 0$ and $T_2(r, z, \tau)$ in the domain $r_0 < r < \infty$, $z \geqslant 0$, $\tau > 0$.

The problem formulated reduces to solving the following system of differential equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right)+\frac{\partial^2 T_1}{\partial z^2}=\frac{1}{a}\frac{\partial T_1}{\partial \tau}\quad (\tau>0,\ z\geqslant 0,\ 0\leqslant r<r_0),$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right)+\frac{\partial^2 T_2}{\partial z^2}=\frac{1}{a}\frac{\partial T_2}{\partial \tau}\quad (\tau>0,\ z\geqslant 0,\ r_0<r<\infty) \tag{1}$$

under the initial

$$T_1(r,\ z,\ 0)=T_2(r,\ z,\ 0)=T_0=\mathrm{const} \tag{2}$$

and the boundary conditions

$$-\lambda\frac{\partial T_1(r,\ 0,\ \tau)}{\partial z}=q(\tau), \tag{3}$$

$$\frac{\partial T_2(r,\ 0,\ \tau)}{\partial z}=0, \tag{4}$$

$$\frac{\partial T_1(0,\ z,\ \tau)}{\partial r}=0, \tag{5}$$

$$T_2(\infty,\ z,\ \tau)=T_0, \tag{6}$$

$$\frac{\partial T_1(r,\ \infty,\ \tau)}{\partial z}=\frac{\partial T_2(r,\ \infty,\ \tau)}{\partial z}=\frac{\partial T_2(\infty,\ z,\ \tau)}{\partial r}=0, \tag{7}$$

$$T_1(r_0,\ z,\ \tau)=T_2(r_0,\ z,\ \tau), \tag{8}$$

$$\frac{\partial T_1(r_0,\ z,\ \tau)}{\partial r}=\frac{\partial T_2(r_0,\ z,\ \tau)}{\partial r}. \tag{9}$$

Let us examine two cases.

1. A constant specific heat flux q_0 = const is delivered through a circle of radius r_0. The boundary condition (3) is written as follows:

$$-\lambda \frac{\partial T_1(r,\ 0,\ \tau)}{\partial z} = q_0. \tag{10}$$

Applying the Fourier cosine and the Laplace transforms to (1), (2), (4)-(9), (10), we obtain the solution of the system of differential equations (1) in the following form:

For the domain $0 \leqslant r < r_0,\ z \geqslant 0,\ \tau > 0$

$$T_1(r,\ z,\ \tau) - T_0 = \frac{2q_0}{\lambda}\sqrt{a\tau}\ \mathrm{ierfc}\,\frac{z}{2\sqrt{a\tau}} - \frac{q_0 r_0}{\lambda} \times$$
$$\times \frac{1}{\pi^2 i} \int\limits_0^\infty \int\limits_{\sigma - i\infty}^{\sigma + i\infty} \frac{I_0(r\sqrt{p^2 + s/a})\,K_1(r_0\sqrt{p^2 + s/a})\cos pz \exp(s\tau)\,ds\,dp}{s\sqrt{p^2 + s/a}}; \tag{11}$$

and for the domain $r_0 < r < \infty,\ z \geqslant 0,\ \tau > 0$

$$T_2(r,\ z,\ \tau) - T_0 = \frac{q_0 r_0}{\lambda}\frac{1}{\pi^2 i} \int\limits_0^\infty \int\limits_{\sigma - i\infty}^{\sigma + i\infty} \frac{I_1(r_0\sqrt{p^2 + s/a})\,K_0(r\sqrt{p^2 + s/a})\cos pz \exp(s\tau)\,ds\,dp}{s\sqrt{p^2 + s/a}}, \tag{12}$$

where $I_0(x)$ and $I_1(x)$ are the modified Bessel functions of the first kind of zeroth and first orders; $K_0(x)$ and $K_1(x)$ are the modified Bessel functions of the second kind of zeroth and first orders, ierfc x is the multiple probability integral. The contour integration in the solutions (11) and (12) is performed (in the complex variable s) along the line Re s = σ.

Consider the particular case of the solution (11) on the axis $z \geqslant 0$ (for r = 0), it is easy to obtain an expression for the excess temperature $\Delta T_1(0,\ z,\ \tau) = T_1(0,\ z,\ \tau) - T_0$ in the following form:

$$\Delta T_1(0,\ z,\ \tau) = \frac{2q_0\sqrt{a\tau}}{\lambda}\ \mathrm{ierfc}\,\frac{z}{2\sqrt{a\tau}} - AW_{-\frac{3}{4},\ \frac{1}{4}}\left(\frac{r_0^2 + z^2}{4a\tau}\right), \tag{13}$$

where $W_{-3/4,\ 1/4}(r_0^2 + z^2/4a\tau)$ is the Whittaker function [6]

$$A = \frac{q_0\sqrt{2}}{\lambda\sqrt{\pi}}(r_0^2 + z^2)^{-1/4}(a\tau)^{3/4}\exp\left(-\frac{r_0^2 + z^2}{8a\tau}\right).$$

The solution (13) corresponds to the solution obtained in [11] if the Whittaker function is expressed in terms of the multiple probability integral

$$W_{-\frac{3}{4},\ -\frac{1}{4}}(x^2) = 2\sqrt{\pi x}\exp(1/2\,x^2)\ \mathrm{ierfc}\,x. \tag{14}$$

In our case $x = 1/2\sqrt{(r_0^2 + z^2)/a\tau}$.

The excess temperature $\Delta T_1(0,\ 0,\ \tau) = T_1(0,\ 0,\ \tau) - T_0$ at the center of the circle (r = 0, z = 0) can, according to (13) and (14), be determined from the expression

$$\Delta T_1(0,\ 0,\ \tau) = \frac{2q_0}{b}\sqrt{\tau}\left(\frac{1}{\sqrt{\pi}} - \mathrm{ierfc}\,\frac{r_0}{2\sqrt{a\tau}}\right), \tag{15}$$

where

$$b = \frac{\lambda}{\sqrt{a}} \tag{16}$$

is the coefficient of thermal activity ($\mathrm{W \cdot sec^{1/2}/m^2 \cdot K}$).

The determination of a and b from (15) is possible by different methods for fixed values of q_0, r_0 and a known dependence $\Delta T_1(0,\ 0,\ \tau) = f(\tau)$. In this paper, it is proposed to determine the thermal activity of a body at times when the argument $r_0/2\sqrt{a\tau} \to \infty$. We then have from (15)

$$b = \frac{2q_0\sqrt{\tau}}{\Delta T_1(0,\ 0,\ \tau)\sqrt{\pi}}. \tag{17}$$

The thermal diffusivity is determined for the value of (17) already known by starting from the equation

$$\operatorname{ierfc}\frac{1}{2\sqrt{\mathrm{Fo}}}=Y, \tag{18}$$

where

$$Y=\frac{1}{\sqrt{\pi}}-\frac{\Delta T_1(0,\ 0,\ \tau)\,b}{2q_0\sqrt{\tau}}.$$

Detailed tables exist [4, 9] for the function (18), from which values of the Fourier criterion are determined for known values of Y. The thermal diffusivity coefficient is computed from the value of Fo

$$a=\frac{r_0^2}{\tau}\,\mathrm{Fo}. \tag{19}$$

After b and a have been determined, the heat-conduction coefficient is easily expressed from (16).

2. A thermal flux varing in time according to the law $q(\tau) = B/\sqrt{\tau}$, where B = const ($\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\sec^{1/2}$) is the probe (circle) constant, is delivered through a circle of radius r_0. The boundary condition (3) is written as follows

$$-\lambda\frac{\partial T_1(r,\ 0,\ \tau)}{\partial z}=\frac{B}{\sqrt{\tau}}. \tag{20}$$

The particular solution $\Delta T_1^*(0,\ z,\ \tau) = T_1(0,\ z,\ \tau) - T_0$ in the domain $0 \leqslant r < r_0$, $z > 0$, $\tau > 0$ on the axis $z \geqslant 0$ ($r = 0$) for the system of differential equations (1) with the initial and boundary conditions (2), (4)-(9), (20) will have the form

$$\Delta T_1^*(0,\ z,\ \tau)=\frac{B\sqrt{\pi a}}{\lambda}\left(\operatorname{erfc}\frac{z}{2\sqrt{a\tau}}-\operatorname{erfc}\frac{\sqrt{r_0^2+z^2}}{2\sqrt{a\tau}}\right). \tag{21}$$

From the solution (21) for the center of the circle with z = 0, the excess temperature $\Delta T_1^*(0,\ 0,\ \tau) = T_1(0,\ 0,\ \tau) - T_0$ equals

$$\Delta T_1^*(0,\ 0,\ \tau)=\frac{B\sqrt{\pi}}{b}\operatorname{erf}\frac{r_0}{2\sqrt{a\tau}}, \tag{22}$$

where erf x is the Gauss error function.

By using expression (22) at the initial times when the argument $r_0/2\sqrt{a\tau} \to \infty$ [here $\operatorname{erf}(r_0/2\sqrt{a\tau}) \to 1$], we obtain a formula to compute the thermal activity of the body

$$b=\frac{B\sqrt{\pi}}{\Delta T_1^*(0,\ 0,\ \tau)}. \tag{23}$$

Values of the function

$$\operatorname{erf}\frac{1}{2\sqrt{\mathrm{Fo}}}=Y^*, \tag{24}$$

where

$$Y^*=\frac{b\Delta T_1^*(0,\ 0,\ \tau)}{B\sqrt{\pi}}, \tag{25}$$

are also tabulated [9]. Therefore, appropriate values of the Fourier criterion can always be found for a given time τ from values of Y^* known from experiment. By knowing Fo we determine a and λ from (19) and (16), respectively.

If the density of the substance under investigation γ (kg/m^3) is known, then the specific heat can be determined for values of the thermal conductivity and thermal diffusivity already known by using the formula

$$c=\frac{\lambda}{a\gamma}. \tag{26}$$

Both the considered methods for determining the thermophysical characteristics assume the presence of two stages of experiment, where the temperature field development in the first stage, when the argument of the Kramp function $r_0/2\sqrt{a\tau} \to \infty$, is described by expressions

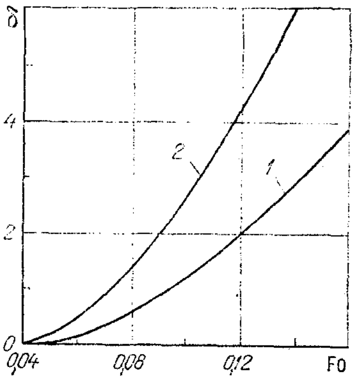

Fig. 1. Graph of the deviation δ (%) of the two-dimensional excess temperature from the one-dimensional excess temperature as a function of the Fourier number Fo.

corresponding to known solutions of the one-dimensional classical boundary-value heat-conduction problem for a half-space with boundary conditions of the first or second kind [9].

To determine the time for performing the experiment, the deviation of the two-dimensional excess temperature from the one-dimensional was computed as a function of Fo. Computations were by means of the formula

$$\delta(\mathrm{Fo}) = \frac{T_{\mathrm{one}} - T_{\mathrm{two}}}{T_{\mathrm{one}}} \cdot 100\%,$$

where

$$\left.\begin{aligned} T_{\mathrm{one}} &= \frac{2q_0 r_0}{\lambda}\sqrt{\mathrm{Fo}}, \\ T_{\mathrm{two}} &= \frac{2q_0 r_0 \sqrt{\mathrm{Fo}}}{\lambda}\left(\frac{1}{\sqrt{\pi}} - \mathrm{ierfc}\,\frac{1}{2\sqrt{\mathrm{Fo}}}\right), \end{aligned}\right\} \text{ for } \quad q(\tau) = q_0 = \mathrm{const},$$

$$\left.\begin{aligned} T_{\mathrm{one}} &= \frac{B\sqrt{\pi}}{b}, \\ T_{\mathrm{two}} &= \frac{B\sqrt{\pi}}{b}\,\mathrm{erf}\,\frac{1}{2\sqrt{\mathrm{Fo}}}, \end{aligned}\right\} \text{ for } \quad q(\tau) = \frac{B}{\sqrt{\tau}}.$$

Results of the computations are represented in the Fig. 1. The time τ corresponding to a definite value of Fo is determined from curves 1 or 2 prior to which (17) or (23), respectively, can be used with sufficient accuracy.

The determination of the thermal diffusivity coefficient from (18) or (24), (25) is evidently possible in later stages of the experiment when the two-dimensionality of the temperature field development is already felt considerably. The influence of the shallowness of the curves ierfc x or erf x and the accuracy of the values of these functions on the accuracy of determining the values of the argument x should be taken into account in selecting the time of onset and duration of this stage.

NOTATION

r, z, coordinates; τ, time; r_0, radius of the heating spot; q_0, $q(\tau)$, constant and variable specific heat flux in the heating spot; T_0, initial temperature; $T_1(r, z, \tau)$, $T_2(r, z, \tau)$, temperature in an arbitrary point of the domain ($0 \le r < r_0$, $z \ge 0$) and the domain ($r_0 < r < \infty$, $z \ge 0$), respectively; ΔT_1, ΔT_1^*, excess temperatures for cases 1 and 2; a, λ, b, c, γ, thermal diffusivity, heat condution, thermal activity, specific heat, and density, respectively; $\mathrm{Fo} = a\tau/r_0^2$, Fourier criterion; B, probe (circle) constant.

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THERMAL CONDUCTIVITY OF KETONES AS A FUNCTION OF TEMPERATURE AND PRESSURE

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Results are presented from an experimental study of thermal conductivity of ketones in the liquid and gaseous states at various temperatures and pressures, including the range of critical and supercritical state parameters.

Analysis of published studies of ketone thermal conductivity shows that this property has been studied mainly as a function of temperature at atmospheric pressure [1, 2]. The results of various studies [2-5] of acetone thermal conductivity versus temperature diverge both qualitatively and quantitatively. At 293°K this divergence is 4%, while at 313°K it reaches 12%.

In the present study measurements were made of the thermal conductivity of liquid and gaseous dimethylketone (acetone), methylethylketone, methylbutylketone, and dipropylketone over the temperature range of 285.9-713.3°K at pressures of $(0.98-490)\cdot 10^5$ Pa. The measurements were performed by the cylindrical calorimeter regular thermal regime method [6, 7]. In the liquid phase the thickness of the layer studied was 0.625 and 0.55 mm, while in the gas phase thicknesses of 0.36 and 0.43 mm were used. The temperature head at the specimen boundary varied from 1.7 to 0.5°K.

Thermal conductivity measurements performed with differing temperature differentials at the specimen boundaries revealed that there was no convective heat exchange in the experiments, as evidenced by the good agreement of all the experimental data. For all measurements the product of the Grashof and Prandtl numbers Gr•Pr was less than 1000, which also confirmed the absence of any convective effect on the thermal conductivity values.

Comparison of the present experimental data on thermal conductivity of liquid ketones at atmospheric pressure with the recommended values of [1] revealed that they coincide to an accuracy of 1-2%. The maximum relative uncertainty of the measurements did not exceed 2.5%, except for the critical region where it increased somewhat to 3-4.5%.

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